2020

CHEMISTRY — HONOURS

Paper : SEC-A-1

(Mathematics and Statistics for Chemists)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question number*** **1** and ***any*** **12** ***(twelve)*** questions from the rest.

1×20

1. (a) Which of the following function is neither even nor odd?

(i) Sin x (ii) Cos x (iii) e^x (iv) $\left(e^x + e^{-x}\right)/2$

(b) What is the relation between *error function* of Z and the *co-error function* of Z?

(c) The Gaussian distribution is given by : $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-x^2/2\sigma^2}$, where $-\infty \leqslant x \leqslant +\infty$.

So the standard deviation in x is

(i) σ (ii) $\sqrt{\sigma}$ (iii) σ^2 (iv) 0

(d) If $\left|\vec{p}\right| = 10$, $\left|\vec{q}\right| = 1$ and $\vec{p} \cdot \vec{q} = 6$, then $\left|\vec{p} \times \vec{q}\right|$ is

(i) 11 (ii) 8 (iii) 10 (iv) 5

(e) In a practical examination a student has to perform two experiments from a set of ten different experiments (to be decided by lottery). How many different combinations are possible?

(f) An ideal coin is tossed three (3) times in succession. Find the probability of getting exactly one head.

(g) Which type of error, 'systematic' or 'random' is associated with the Gaussian distribution?

(h) In an experiment involving 'N' number of observations, what will be the value of the 'degrees of freedom'?

(i) If the coefficient of correlation between two variables X and Y is 0.28, their co-variance is + 7.6. If the variance of X is 9, find the standard deviation of Y.

(j) Find the 'integrating factor' (by inspection) which makes the inexact equation $2x\,dy + y\,dx = 0$ an exact one.

(k) State the order and degree of the differential equation

$$\left(\frac{\partial^3 y}{\partial x^3}\right)^2 + 4x\left(\frac{\partial y}{\partial x}\right)^2 + y\left(\frac{\partial y}{\partial x}\right) = 0$$

(l) Identify the multi-valued function among the following :

$\cot x$, $\sqrt{x}$, $\log x$.

(m) Write down the Laplace transformation of $e^{-2t}\,\mathrm{Sin}(4t)$.

(n) For $Z = f(x, y)$, mention the criterion for dZ to be an exact differential.

(o) Write down the Fourier transformation for $f(x) = x$, in the interval $(-L\ ,\ +L)$, considering 'x' as an odd function.

(p) Give the cross-product of the following vectors :

$\vec{A} = 3\vec{i} - 3\vec{j} + \vec{k}$; $B = 4\vec{i} + 9\vec{j} + 2\vec{k}$

(q) For Maxwell's distribution of molecular velocity in one-dimension $\frac{1}{N}\frac{dN_x}{dv_x} = A \cdot e^{-mv_x^2/2kT}$,

justify that $\vec{v}_x = 0$, using qualitative arguments.

(r) Mention the percentage of the areas corresponding to the interval ($\mu \pm \sigma$) and ($\mu \pm 2\sigma$) in case of a normal distribution.

(s) What is the physical significance of the variance?

(t) For a normal distribution which of the following is true?

(i) mean = median = mode

(ii) mean = median ≠ mode

(iii) mean ≠ median = mode

2. Maxwell's kinetic energy distribution in one-dimension is given by :

$$\frac{dN\epsilon}{N} = \left(\frac{1}{\pi kT}\right)^{1/2} \cdot \epsilon^{-1/2} \cdot e^{-\epsilon/kT}\ d\epsilon\ ,$$

where the terms have their usual significance.

Using gamma function, evaluate $\left\langle \epsilon^2 \right\rangle$. 5

3. (a) What do you mean by '$f(x)$ is continuous at $x = a$' ? Hence justify that $f(x) = 1/x^2$ is *not* continuous at $x = 0$.

(b) Find the limit $\underset{x \to 0}{Lt} \dfrac{1 - e^x}{x}$. 3+2

4. (a) Give an outline of the method used to find the most probable quantity (C_{mp}) from a given distribution function $f(c)$.

(b) What do you mean by a point of inflection on the curve $f(x)$? Demonstrate using the van der Waals equation of state. 2+3

5. Find the Fourier series for the function

$$f(t) = \begin{cases} -1, & \text{for } -\pi < t < -\pi/2 \\ 0, & \text{for } -\pi/2 < t < \pi/2 \\ +1, & \text{for } \pi/2 < t < \pi \end{cases}$$

where $f(t) = \dfrac{a_0}{2} + \sum\limits_{n=1}^{\infty} a_n \operatorname{Cos}(nt) + \sum\limits_{n=1}^{\infty} b_n \operatorname{Sin}(nt)$. 5

6. Solve the differential equation for the *consecutive reaction* $A \xrightarrow{K_1} B \xrightarrow{K_2} C$ (both first-order).

$\dfrac{d[B]}{dt} = K_1 [A] - K_2 [B]$, and find the values of $[A]$, $[B]$ and $[C]$ at time 't'.

Given $[A]_{t=0} = [A]_0$, $[B]_0 = 0 = [C]_0$ 5

7. Check the equations whether they are consistent or not :

$2x - 3y + 5z = 1$

$3x + y - z = 2$

$x + 4y - 6z = 1$

and solve them by the matrix method. 5

8. The homogenity of the chloride level in a water sample from a lake was tested by analyzing portions drawn from the top and bottom of the lake, with the following results, expressed in ppm of chloride,

Top	26.30	26.43	26.28	26.19	26.49
Bottom	26.22	26.32	26.20	26.11	26.42

Apply the *paired t-test* and determine whether there is a significant difference between the top and bottom values at 95% confidence level. 5

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9. Apply the Q-test to the following data sets to determine whether the sets should be retained or rejected at the 95% confidence level :

(A) 41.27, 41.61, 41.84, 41.70

(B) 7.295, 7.284, 7.388, 7.292 5

10. (a) The percentage of an additive in gasoline was measured six (6) times with the following results : 0.13, 0.12, 0.16, 0.17, 0.20, 0.11 %.

Find the 90% and 99% confidence intervals for the percentage of the additive.

(b) For a sample having a mean $\left(\bar{x}\right)$ of 10.0 and the probability of the population mean (μ) being in the region 10.0 ± 1, is 0.9. Find the confidence interval (CI), confidence limit and confidence level. 3+2

11. The following results were obtained in the replicate analysis of a blood sample for its lead content : 0.752, 0.756, 0.753, 0.751, 0.760 (ppm of Pb). Calculate the mean, the standard deviation, relative standard deviation, coefficient of variance and spread of this set of data. 5

12. Find the *linear regression* parameters from the following data :

$$\sum_i x_i = 24\,,\quad \sum_i y_i = 44\,,\quad \sum x_i\, y_i = 306\,,$$

$$\sum_i x_i^2 = 164,\quad \sum y_i^2 = 574,\quad n = 4.$$ 5

13. (a) Suggest a suitable transformation of the function

$y = a\, e^{-bx}$, to find the parameters a and b by the 'linear least squares' method.

(b) Justify that the 'least squares best fit' straight line passes through the 'average point' $\left(\bar{x}, \bar{y}\right)$. 2+3

14. For the vectors : $\vec{A} = 2\vec{i} + 3\vec{j} + \vec{k}$ and $\vec{B} = 3\vec{i} + 4\vec{j} - \vec{k}$, find the following :

(a) $|A|$ and $|B|$

(b) $|C|$, when $\vec{C} = \vec{B} - \vec{A}$

(c) $\vec{A} \cdot \vec{B}$

(d) The angle between $\vec{A}$ and $\vec{B}$. 5

15. (a) State the Maclaurin series for $f(x)$. Can you apply the series for $f(x) = \log x$ around $x = 0$? Justify your answer.

(b) Use the Maclaurin series to find the expansion of $f(x) = ln\,(1 - x)$, up to three terms. 2+3